

TOPIC

- **Frustum**
- **Sums based on Frustum of Cone**
- **Sum based on Frustum of Cone (PART II)**



SURFACE AREAS AND VOLUMES

- **Frustum**

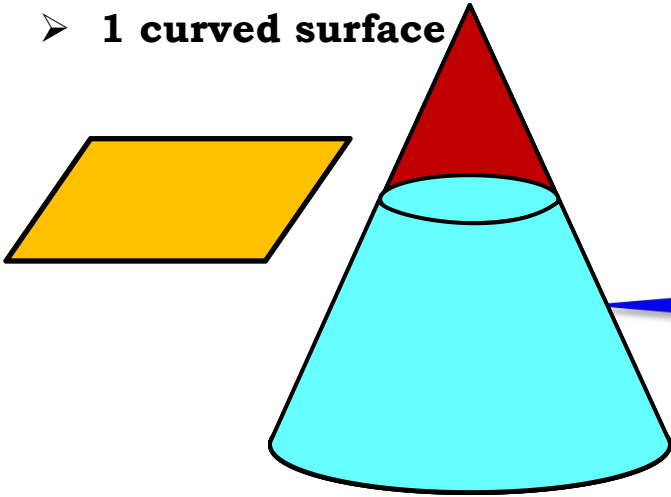
Frustum

3 surfaces :

- 2 circular surfaces
- 1 curved surface

Cone

Frustum



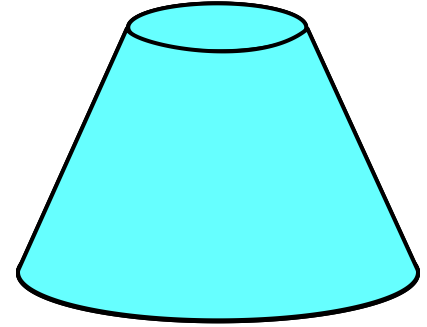
Formulae

Curved surface area = $\pi(r_1 + r_2)l$

Total surface area = **A(two circular + C.S.A. surfaces)**

$$= \pi r_1^2 + \pi r_2^2 + \pi(r_1 + r_2)l$$

$$\text{Volume} = \frac{1}{3} \pi (r_1^2 + r_2^2 + r_1 \cdot r_2)h$$





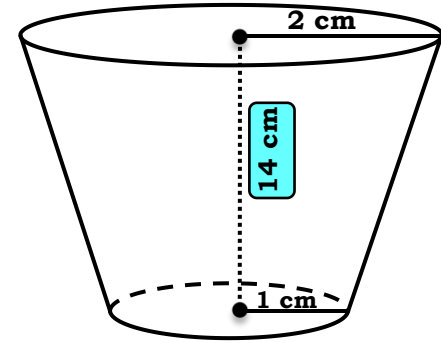
SURFACE AREAS AND VOLUMES

- **Sums based on Frustum of Cone**

Q. A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.

Sol. $r_1 = \frac{4}{2} = 2 \text{ cm}$

$r_2 = \frac{2}{2} = 1 \text{ cm}$



$$\begin{aligned} \text{Volume of frustum} &= \frac{1}{3} \pi h (r_1^2 + r_2^2 + r_1 r_2) \\ &= \frac{1}{3} \times \frac{22}{7} \times 14 \times (2^2 + 1^2 + \underline{2 \times 1}) \end{aligned}$$

Let the radius of the bottom circular face be r_2

$$\begin{aligned} &= \frac{22}{7} \times 14 \times (4 + 1 + 2) \\ &= \frac{22}{3} \times \cancel{14}^2 \times 7 \\ &= \frac{1}{3} \times 22 \times 2 \times 7 \\ &= \frac{308}{3} = 102.67 \text{ cm}^3 \end{aligned}$$

\therefore Capacity of the glass is 102.67 cm^3

Q. The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Sol. $2\pi r_1 = 18$... (i)
 $2\pi r_2 = 6$... (ii)

Adding (i) & (ii)

$$2\pi r_1 + 2\pi r_2 = 18 + 6$$

$$2\pi r_1 + 2\pi r_2 = 24$$

$$2\pi (r_1 + r_2) = 24$$

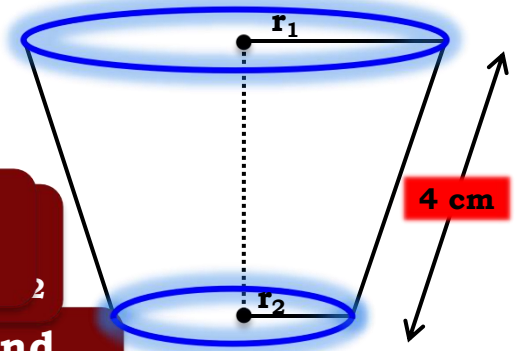
$$\therefore r_1 + r_2 = \frac{24}{2\pi}$$

$$\therefore r_1 + r_2 = \frac{12}{\pi}$$

To find : $r_1 + r_2$

Let the radius of the top circular face be r_1

What is the formula to find curved surface area of frustum?
 $\pi l (r_1 + r_2)$



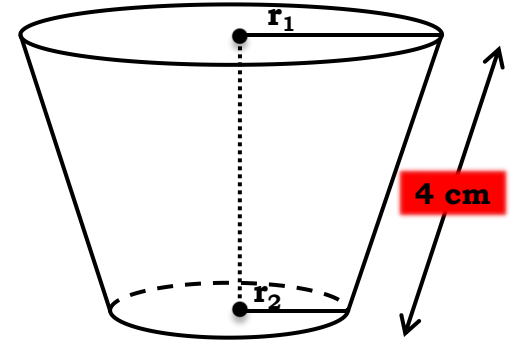
Q. The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Sol.

$$r_1 + r_2 = \frac{12}{\pi}$$

$$\begin{aligned}\text{CSA of the frustum} &= \pi l (r_1 + r_2) \\ &= \cancel{\pi} \times 4 \times \frac{12}{\cancel{\pi}} \\ &= 4 \times 12 \\ &= 48 \text{ cm}^2\end{aligned}$$

∴ Curved surface area of the frustum is 48 cm²





SURFACE AREAS AND VOLUMES

- **Sum based on Frustum of cone**

Q. A *fez*, the cap used by the Turks, is shaped like the frustum of a cone. If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 15 cm, find the area of material used in making it.

$$\pi l (r_1 + r_2)$$

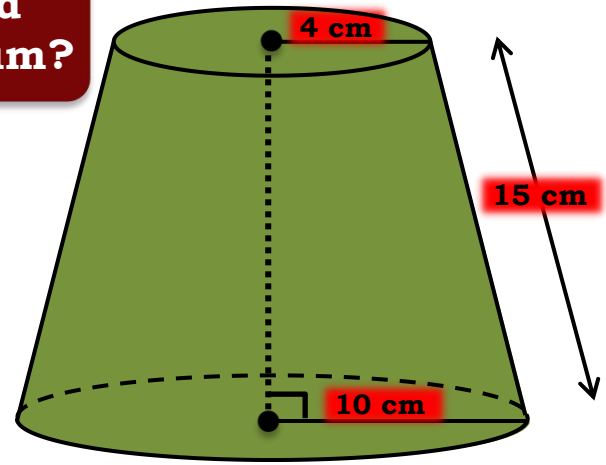
$$\pi r_2^2$$

Area of material used = CSA of (Frustum) + Area of top

Sol. $r_1 = 10 \text{ cm}$, $r_2 = 4 \text{ cm}$, $l = 15 \text{ cm}$

What is the formula to find curved surface area of frustum?

What is the formula to find area of top?



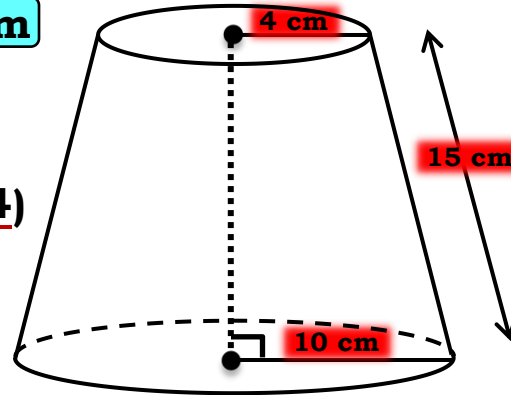
Q. A *fez*, the cap used by the Turks, is shaped like the frustum of a cone. If its radius on the open side is 10 cm, radius at the upper base is 4 cm, and its slant height is 15 cm, find the area of material used in making it.

Area of material used = CSA of (Frustum) + Area of top

Sol. $r_1 = 10$ cm, $r_2 = 4$ cm, $l = 15$ cm

$$\begin{aligned}
 \text{CSA of (Frustum)} &= \pi l (r_1 + r_2) \\
 &= \frac{22}{7} \times 15 \times (10 + 4) \\
 &= \frac{22}{7} \times 15 \times 14 \\
 &= 22 \times 15 \times 2
 \end{aligned}$$

\therefore CSA of (Frustum) = 660 cm²

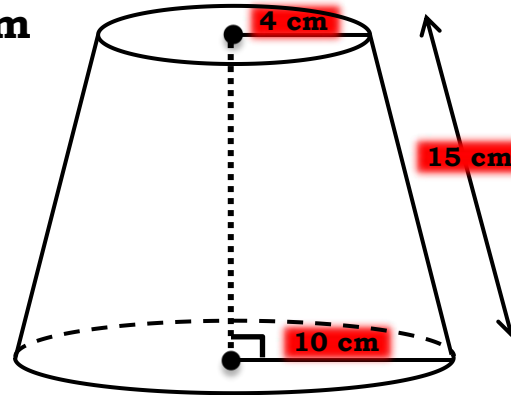


Q. A fez, the cap used by the Turks, is shaped like the frustum of a cone. If its radius on the open side is 10 cm, radius at the upper base is 4 cm, and its slant height is 15 cm, find the area of material used in making it.

Area of material used = CSA of (Frustum) + Area of top

Sol. $r_1 = 10$ cm, $r_2 = 4$ cm, $l = 15$ cm

$$\begin{aligned}
 \text{Area of top} &= \pi r_2^2 \\
 &= \frac{22}{7} \times (4)^2 \\
 &= \frac{22}{7} \times 16 \\
 \therefore \text{Area of top} &= \frac{352}{7} \text{ cm}^2
 \end{aligned}$$



Q. A *fez*, the cap used by the Turks, is shaped like the frustum of a cone. If its radius on the open side is 10 cm, radius at the upper base is 4 cm, and its slant height is 15 cm, find the area of material used in making it.

Area of material used = CSA of (Frustum) + Area of top

Sol. $r_1 = 10$ cm, $r_2 = 4$ cm, $l = 15$ cm

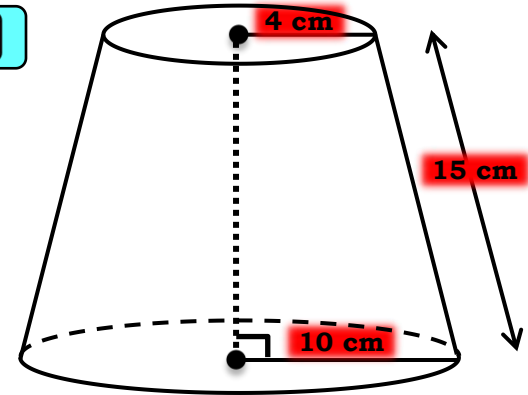
Area of material used = **CSA of (Frustum) + Area of top**

$$\begin{aligned}
 &= \\
 &= 660 + \frac{352}{7} \\
 &= \frac{4972}{7} \\
 &= 710.28 \text{ cm}^2
 \end{aligned}$$

CSA of (Frustum) = 660 cm²

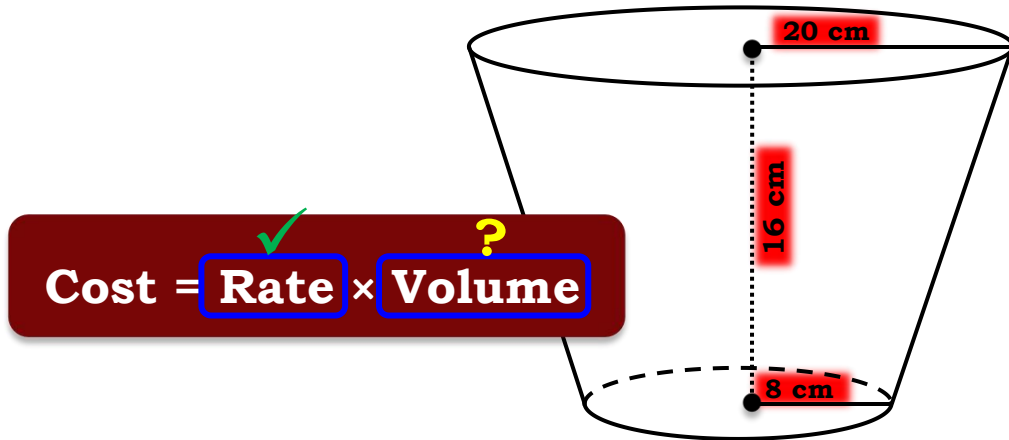
∴ Total area is 710.28 cm²

Area of top = $\frac{352}{7}$ cm²



Q. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of Rs 20 per litre. Also find the cost of metal sheet used to make the container, if it costs Rs 8 per 100 cm². (Take $\pi = 3.14$)

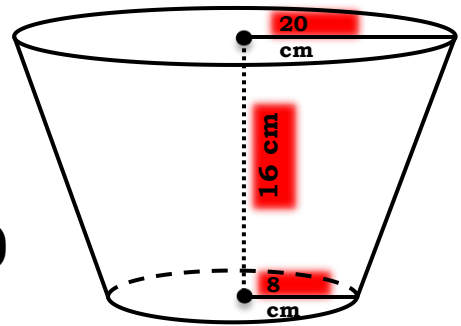
Sol. $h = 16$ cm, $r_1 = 8$ cm, $r_2 = 20$ cm To find : Vol.



Q. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively.

Find the cost of the milk which can completely fill the container, at the rate of Rs 20 per litre. Also find the cost of metal sheet used to make the container, if it costs Rs 8 per 100 cm². (Take $\pi = 3.14$)

Sol. $h = 16$ cm, $r_1 = 8$ cm, $r_2 = 20$ cm To find : Vol.



$$\text{Vol. of frustum of a cone} = \frac{1}{3} \pi h (r_1^2 + r_2^2 + r_1 r_2)$$

$$= \frac{1}{3} \times 3.14 \times 16 \times (8^2 + 20^2 + 8 \times 20)$$

What is the formula to find volume of the frustum of cone?

$$= \frac{1}{3} \times 3.14 \times 16 \times (64 + 400 + 160)$$

$$= \frac{1}{3} \times \frac{314}{100} \times 16 \times 208$$

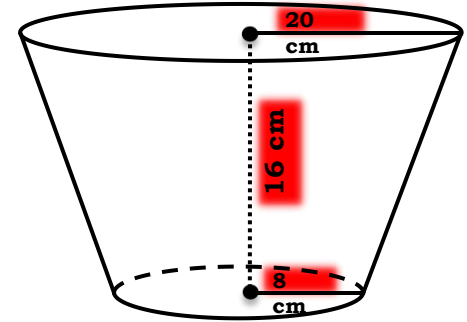
$$\text{Vol. of frustum of a cone} = \frac{314 \times 16 \times 208}{100}$$

Q. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of Rs 20 per litre. Also find the cost of metal sheet used to make the container, if it costs Rs 8 per 100 cm². (Take $\pi = 3.14$)

Sol. $h = 16 \text{ cm}$, $r_1 = 8 \text{ cm}$, $r_2 = 20 \text{ cm}$ **To find : Vol.**

$$\begin{aligned} \text{Vol. of frustum of a cone} &= \frac{314 \times 16 \times 208}{100} \\ &= \frac{1044992}{100} \\ &= 10449.92 \text{ cm}^3 \\ &= \frac{10449.92}{1000} \text{ litres} \\ &= 10.44992 \text{ litres} \end{aligned}$$

\therefore Vol. of frustum of a cone = 10.45 litres



Q. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of Rs 20 per litre. Also find the cost of metal sheet used to make the container, if it costs Rs 8 per 100 cm². (Take $\pi = 3.14$)

Sol. $h = 16 \text{ cm}$, $r_1 = 8 \text{ cm}$, $r_2 = 20 \text{ cm}$

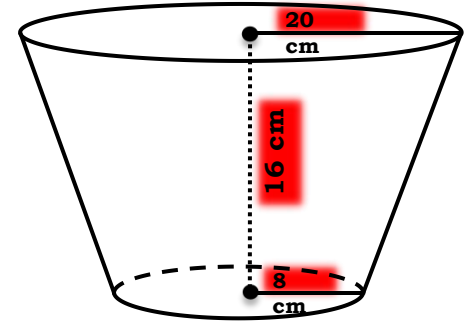
Vol. of frustum of a cone = 10.45 litres

Rate = Rs. 20 per litre

Cost = Rate \times Volume

$$\begin{aligned} \text{Cost} &= \text{Rate} \times \text{Volume} \times 10.45 \\ &= \text{Rs. } 209 \end{aligned}$$

\therefore Cost of the milk which can completely fill the container is Rs. 209





SURFACE AREAS AND VOLUMES

- **Sum based on Frustum of Cone (PART II)**

Q. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of Rs 20 per litre, and the area of metal sheet used to make the container, if it is open at the top. (Take $\pi = 3.14$)

Area of material used = CSA of (Frustum) + Area of base

Sol. $h = 16$ cm, $r_1 = 8$ cm, $r_2 = 20$ cm

$$\therefore l = \sqrt{(r_1 - r_2)^2 + h^2}$$

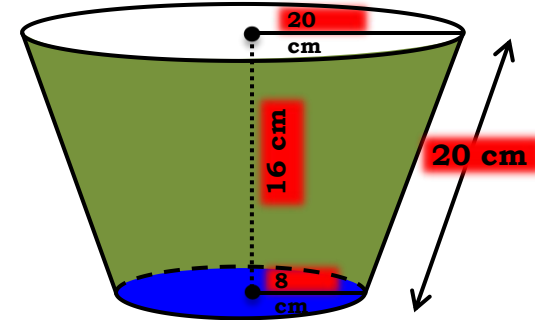
Cost = Rate \times Area

$$= \sqrt{(8 - 20)^2 + (16)^2}$$

What is the formula to find area of base? curved surface area of frustum?

$$= \sqrt{400} \text{ height of the cone}$$

$$\therefore l = 20 \text{ cm}$$



Q. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of Rs 20 per litre. Find the area of metal sheet used to make the container, if it is open from the top. (Take $\pi = 3.14$)

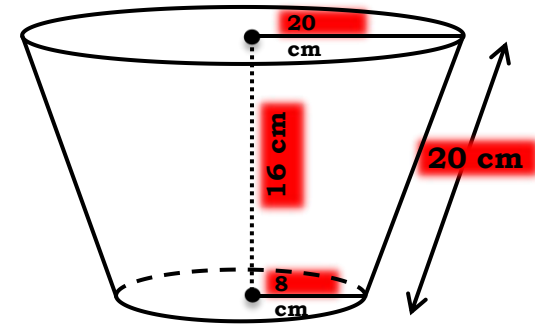
Area of material used = CSA of (Frustum) + Area of base

Sol. $h = 16$ cm, $r_1 = 8$ cm, $r_2 = 20$ cm

Area of material used = CSA of (Frustum) + Area of base

$$\begin{aligned}
 &= \pi l (r_1 + r_2) + \pi r_1^2 \\
 &= \pi \times 20 \times (8 + 20) + \pi \times (8)^2 \\
 &= \pi \times [20 \times 28 + 64] \\
 &= \pi \times [560 + 64] \\
 &= \pi \times 624 \\
 &= 3.14 \times 624
 \end{aligned}$$

\therefore Area of material used = 1959.36 cm²



Q. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of Rs 20 per litre. Also find the cost of metal sheet used to make the container, if it costs Rs 8 per 100 cm². (Take $\pi = 3.14$)

Sol. Area of material used = 1959.36 cm²

Rate = Rs. 8 per 100 cm²

Rate = Rs. $\frac{8}{100}$ per cm²

Cost = Rate × Area

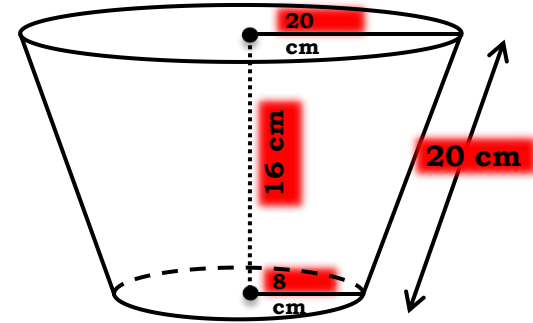
$$= \frac{8}{100} \times 1959.36$$

$$= \frac{15674.88}{100}$$

$$= \text{Rs. } 156.75$$

Cost = Rate × Area

∴ Cost of metal sheet used to make the container is Rs. 156.75

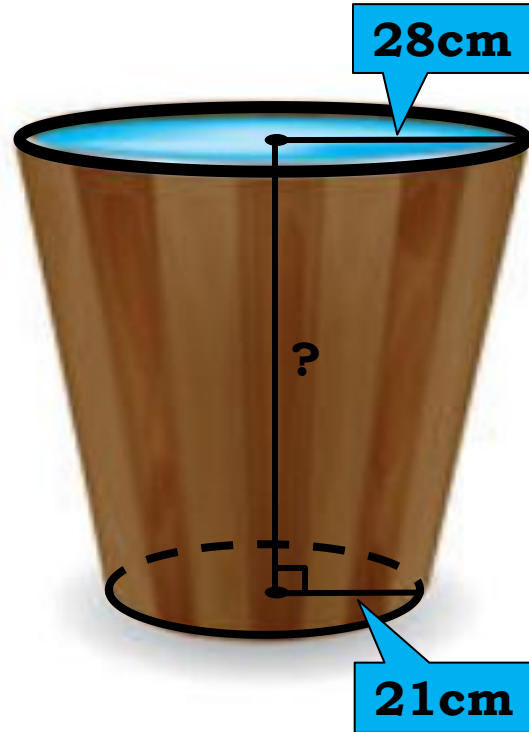




SURFACE AREAS AND VOLUMES

- **Sum based on Frustum of cone**

Q. A bucket in the form of a frustum of a cone and holds 28.490 litres of water. The radii of the top and bottom are 28 cm and 21 cm respectively. Find the height of the bucket.



Q. A bucket in the form of a frustum of a cone and holds 28.490 litres of water. The radii of the top and bottom are 28 cm and 21 cm respectively. Find the height of the bucket.

Sol. Let the height of the bucket be h cm.

$$r_1 = 28 \text{ cm,}$$

$$r_2 = 21 \text{ cm,}$$

Volume of the bucket = 28.490 litres

$$= 28.490 \times 1000$$

Let us convert
litres into cm^3

$$\text{The bucket} = 28490 \text{ cm}^3$$

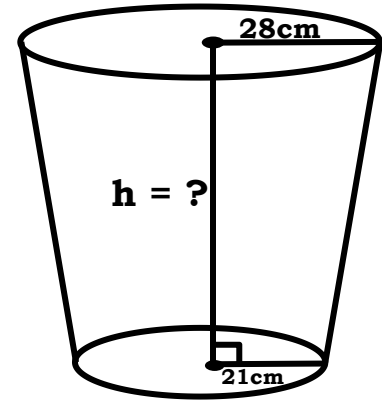
Volume of the frustum

$$= \frac{1}{3} \pi h (r_1^2 + r_1 r_2 + r_2^2)$$

$$28490 = \frac{1}{3} \times \frac{22}{7} \times h \times (28^2 + 28 \times 21 + 21^2)$$

What is the formula to
find volume of the
frustum?

$$\frac{22}{21} \times h \times (784 + 588 + 441)$$



Q. A bucket in the form of a frustum of a cone and holds 28.490 litres of water. The radii of the top and bottom are 28 cm and 21 cm respectively. Find the height of the bucket.

Sol.

$$28490 = \frac{22}{21} \times h \times (784 + 588 + 441)$$

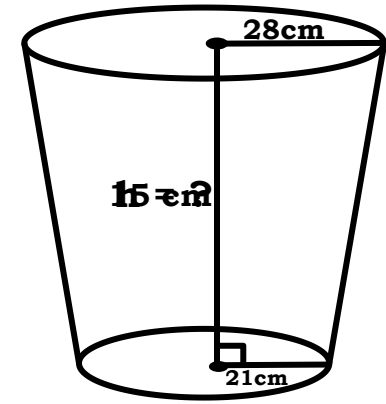
$$\therefore 28490 = \frac{22}{21} \times h \times 1813$$

$$\therefore h = \frac{\overset{2590}{\cancel{28490}} \times \overset{3}{\cancel{21}}}{\underset{259}{\cancel{22}} \times \cancel{1813}}$$

$$\therefore h = \frac{\overset{510}{\cancel{2590}} \times \cancel{3}}{\cancel{2} \times \cancel{259}}$$

$$\therefore h = 15 \text{ cm}$$

\therefore Height of the bucket is 15 cm

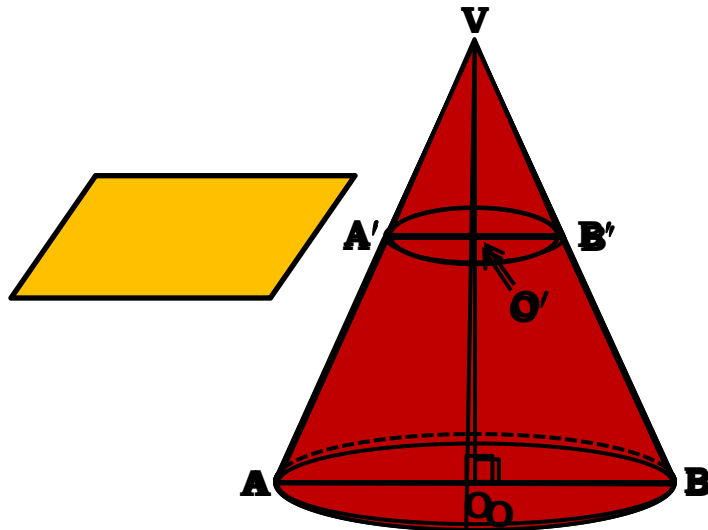




SURFACE AREAS AND VOLUMES

- **Sum based on Frustum of cone**

Q. A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface of the remainder is $\frac{8}{9}$ of the curved surface of the whole cone, find the ratio of the line-segment into which the cone's altitude is divided by the plane.



Q. A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface of the remainder is $\frac{8}{9}$ of the curved surface of the original cone, find the ratio of the line-segment into which the height is divided by the plane.

Consider $\triangle VOA$ and $\triangle VO'A'$ as similar triangles

Sol. Let VAB be a hollow cone of height H , slant height L and base radius R .

Let $VA'B'$ be a small cone of height h , slant height l and base radius r .

To find: $\frac{VO'}{O'O}$ i.e. $\frac{h}{H-h}$

$\triangle VOA$ and $\triangle VO'A'$,

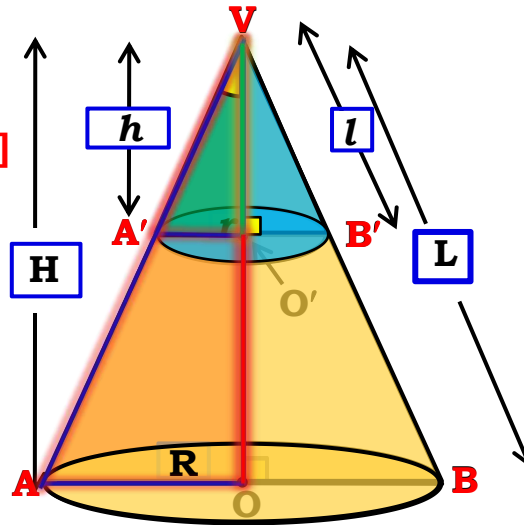
$\angle O'OA = \angle VO - VO'$ [common angle]

$\therefore \angle O'O = H - h$ [\because each 90°]

$\therefore \triangle VOA \sim \triangle VO'A'$ [By AA test]

$$\frac{VO}{VO'} = \frac{OA}{O'A'} = \frac{VA}{VA'} \quad \text{[c.s.s.t.]}$$

$$\frac{H}{h} = \frac{R}{r} = \frac{L}{l} \quad \dots(i)$$



Q. A hollow cone is cut by a plane parallel to the base and the

curved surface of the remainder is $\frac{8}{9}$ of the curved surface of the original cone. Find the ratio of the line-segment cut by the plane.

What is the formula to find curved surface area of the cone?

Sol. CSA of the $ABB'A'$ = CSA of VAB - CSA of $VA'B'$

$$= \pi RL - \pi rl$$

\therefore CSA of the $ABB'A'$ = $\pi (RL - rl)$

CSA of the $ABB'A'$ = $\frac{8}{9} \times$ CSA of VAB

$\therefore \pi (RL - rl) = \frac{8}{9} \times \pi RL$

$\therefore 9(RL - rl) = 8RL$

$\therefore 9RL - 9rl = 8RL$

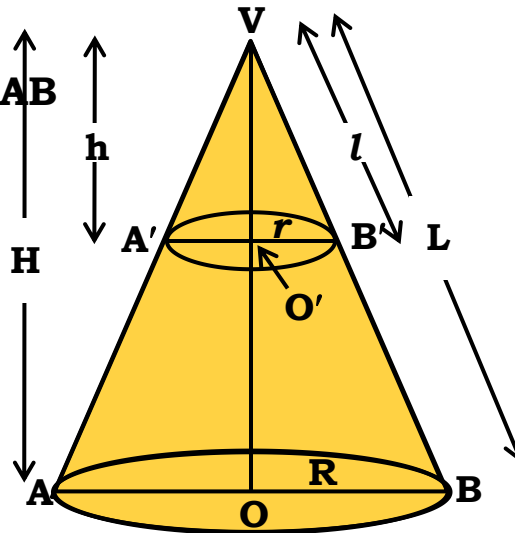
$\therefore 9RL - 8RL = 9rl$

$\therefore RL = 9rl$

$\therefore \frac{RL}{rl} = \frac{9}{1}$

$$\frac{h}{H-h} = ?$$

$$\frac{H}{h} = \frac{R}{r} = \frac{L}{l}$$



Q. A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface of the remainder is $\frac{8}{9}$ of the curved surface of the whole cone, find the ratio of the line-segment into which the cone's altitude is divided by the plane.

Sol. $\frac{RL}{rl} = \frac{9}{1}$

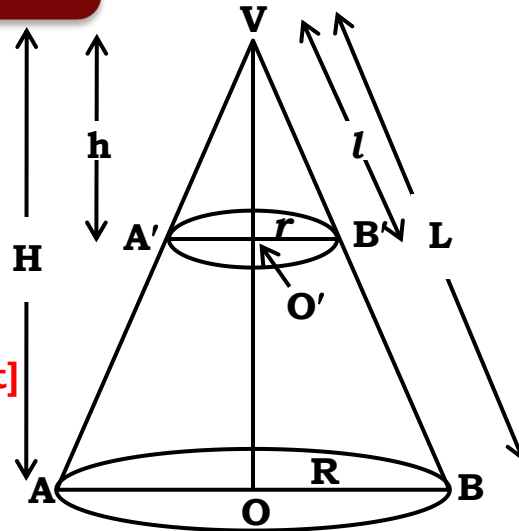
$\therefore \frac{R}{r} \times \frac{L}{l} = \frac{9}{1}$

$\therefore \frac{H}{h} \times \frac{H}{h} = \frac{9}{1}$

$\therefore \frac{H^2}{h^2} = \frac{9}{1}$

$\therefore \frac{H}{h} = \frac{3}{1}$ [Taking square root]

We want ratio in terms of height



$$\frac{h}{H-h} = ?$$

$$\frac{H}{h} = \frac{R}{r} = \frac{L}{l}$$

Q. A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface of the remainder is $\frac{8}{9}$ of the curved surface of the whole cone, find the ratio of the line-segment into which the cone's altitude is divided by the plane.

$$\frac{h}{H-h} = ?$$

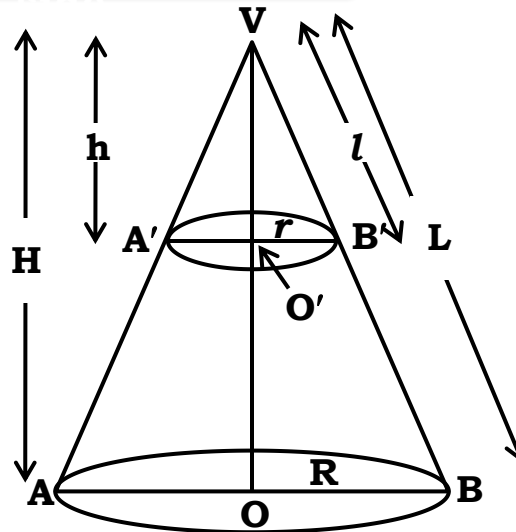
Sol. $\frac{H}{h} = \frac{3}{1}$ [Taking **Subtracting 1 on both** sides]

$$\therefore \frac{H}{h} - 1 = \frac{3}{1} - 1$$

$$\therefore \frac{H-h}{h} = \frac{3-1}{1}$$

$$\therefore \frac{H-h}{h} = \frac{2}{1}$$

$$\therefore \frac{h}{H-h} = \frac{1}{2}$$



Thank You